

Almost Marginal Conditional Stochastic Dominance

Michel M. DENUIT

Université Catholique de Louvain

Rachel J. HUANG

National Taiwan University of Science and
Technology

Larry Y. TZENG

National Taiwan University

Christine WANG

National Taiwan University

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Marginal Conditional Stochastic Dominance

AMCSD

Denuit, Huang,
Tzeng and Wang

- ▶ Shalit and Yitzhaki (1994, MS) introduced the concept of marginal conditional stochastic dominance (MCSD) as a condition under which all risk-averse expected utility maximizer individuals prefer to increase the share of one risky asset over that of another given a portfolio of assets.
- ▶ MCSD is better than the mean-variance (MV) rule since the MV rule requires strong assumptions (such as quadratic utility functions or normally distributed returns) which seldom hold in practice.
- ▶ MCSD has been successfully applied to solve asset allocation problems by several authors, including Clark et al. (2011), Clark and Kassimatis (2012a,b), Shalit and Yitzhaki (2010).

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A Problem of MCSD

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- ▶ Despite the theoretical attractiveness, MCSD rule may create paradoxes in the sense that it fail to distinguish between some risky prospects whereas it is obvious that the vast majority of investors would prefer one over the other.
- ▶ This kind of drawback was first pointed out by Leshno and Levy (2002, MS) on stochastic dominance. They suggested to consider all utility functions after eliminating pathological preferences, keeping only the economically relevant utility functions.

What are your preferences?

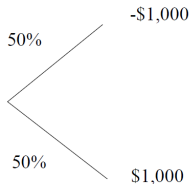
Lottery A1

\$0 for sure

Lottery B1

\$1,000 for sure

Lottery A2



Lottery B2

\$0 for sure

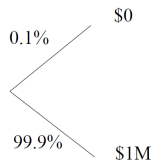
An Example

▶ Lottery Choice

Lottery A3

\$1 for sure

Lottery B3



A risk-averse individual with utility

$$u(x) = \begin{cases} x & \text{if } x \leq 1 \\ 1 & \text{otherwise} \end{cases} \quad \text{would prefer A3 to B3.}$$

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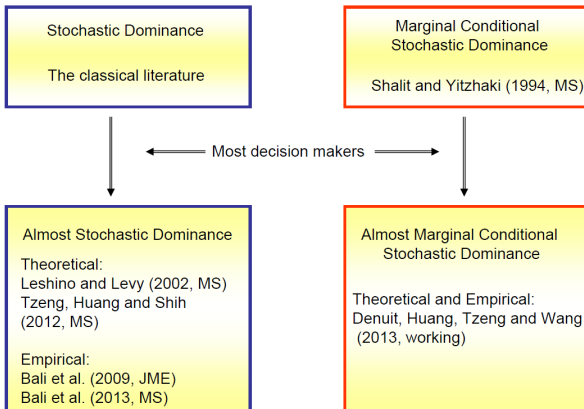
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- ▶ In this paper, MCSD is weakened to ensure that most (but not all) risk-averse decision-makers increase the share of one risky asset over another.
- ▶ This extension of MCSD to AMCSD is inspired from almost stochastic dominance rules introduced by Leshno and Levy (2002, MS), suitably corrected by Tzeng et al. (2012, MS).

Literature Map



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Marginal Conditional Stochastic Dominance

Assumptions

- ▶ Assume that a risk-averse investor with a utility function u holds a portfolio with n risky assets.
- ▶ Let w_0 be the initial wealth, X_i denote the rate of return on risky asset i and α_i be the investment proportion on asset i , $i = 1, 2, \dots, n$.
- ▶ A portfolio α is defined by the shares α_i such that $\sum_{i=1}^n \alpha_i = 1$.
- ▶ The final wealth of the investor is given by $W = w_0 (1 + \sum_{i=1}^n \alpha_i X_i)$.
- ▶ Henceforth, we normalize the initial wealth w_0 to unity so that $W = 1 + \sum_{i=1}^n \alpha_i X_i$.
- ▶ Let R denote the portfolio return, i.e., $R = \sum_{i=1}^n \alpha_i X_i$.

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Marginal Conditional Stochastic Dominance

Objective

- ▶ The goal of the investor is to select the weights to maximize $E[u(W)]$.
- ▶ Given a portfolio α , it is optimal to increase the weight α_k of asset k at the expense of asset j if, and only if,

$$\begin{aligned}
 \frac{dE[u(W)]}{d\alpha_k} &= E\left[u'(W)\frac{dW}{d\alpha_k}\right] \\
 &= E\left[u'(W)\left(X_k + \frac{d\alpha_j}{d\alpha_k}X_j\right)\right] \\
 &= E\left[u'(W)(X_k - X_j)\right] \geq 0 \quad (1)
 \end{aligned}$$

since $\alpha_k + \alpha_j = 0$.

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Condition

- ▶ Shalit and Yitzhaki (1994) proved that for a given portfolio α , asset k dominates asset j for all risk averse agents if and only if

$$E[X_k | R \leq r] \geq E[X_j | R \leq r], \forall r.$$

- ▶ MCSD favors assets performing better in adverse situations (i.e. when the portfolio underperforms $\Leftrightarrow R \leq r$).

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From MCSD to AMCSD

Confined concave utility functions

- ▶ MCSD is based on all the non-decreasing and concave utility functions, that is, on the utility functions in

$$U_2 = \{ \text{utility functions } u \mid u' \geq 0 \text{ and } u'' \leq 0 \}.$$

- ▶ To reveal a preference for most investors, but not for all of them, we restrict U_2 to a subset of it. Specifically, following Leshno and Levy (2002), let us further impose restrictions on the utility function and define

$$U_2^*(\varepsilon) = \left\{ u \in U_2 \mid -u''(x) \leq \inf \{ -u''(x) \} \left(\frac{1}{\varepsilon} - 1 \right) \quad \forall x \right\},$$

where $\varepsilon \in (0, \frac{1}{2})$.

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Condition

► Define

$$B(t) = \left(E[X_k | R \leq t] - E[X_j | R \leq t] \right) F_R(t)$$

$$\Omega = \{t \in [a, b] \mid B(t) < 0\}$$

and Ω^c denote the complement of Ω in $[a, b]$. MCSD requires $B(t) \geq 0$ for all t , that is, $\Omega = \emptyset$. If this is not the case, Ω represents the set of violation for MCSD.

Theorem

Given portfolio α , asset k dominates asset j for all individuals with preferences represented by the utility function $u \in U_2^(\varepsilon)$ if, and only if,*

$$\int_{\Omega} (-B(t) dt) \leq \varepsilon \int_a^b |B(t)| dt \quad (2)$$

and $E[X_k] \geq E[X_j]$.

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An example

- ▶ The distributions of the rates of return for three assets are respectively

$$X_1 = \begin{cases} -10\% \text{ with probability } \frac{1}{2} \\ +15\% \text{ with probability } \frac{1}{2} \end{cases}$$

$$X_2 = \begin{cases} -11\% \text{ with probability } \frac{1}{2} \\ +50\% \text{ with probability } \frac{1}{2} \end{cases}$$

$$X_3 = \begin{cases} -15\% \text{ with probability } \frac{1}{2} \\ +25\% \text{ with probability } \frac{1}{2} \end{cases}$$

- ▶ The weights in the current portfolio are $\alpha_1 = 25\%$, $\alpha_2 = 50\%$ and $\alpha_3 = 25\%$.

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Absolute Concentration Curves (ACCs)

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- ▶ Shalit and Yitzhaki (1994) related MCSD to Absolute Concentration Curves (ACCs) defined as follows. The ACC for asset i with respect to the portfolio α is

$$ACC_i(p) = E[X_i | R \leq F_R^{-1}(p)]$$

where $F_R^{-1}(p)$ is the p th quantile of the distribution function F_R formally defined as

$$F_R^{-1}(p) = \inf\{\xi \in \mathbb{R} | F_R(\xi) \geq p\}.$$

Figure 1 ACCs

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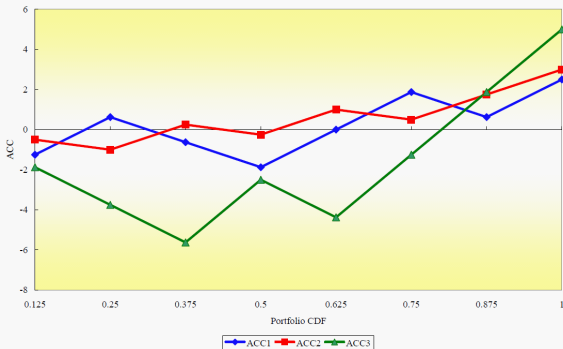
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The criteria of AMCS D

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- ▶ Let us assume that ε is equal to 0.3.

| | 2 vs 1 | 2 vs 3 | 3 vs 1 |
|---|--------|--------|--------|
| Differences in expectations | 17 | 14.5 | 2.5 |
| $\varepsilon \int_a^b B(t) dt - \int_{\Omega} (-B(t)) dt$ | 1.025 | 2.675 | -4 |

- ▶ If $\varepsilon = 0.3$, then Asset 2 AMCS D Asset 1 and Asset 3.
- ▶ If $\varepsilon = 0.1$, then Asset 2 AMCS D Asset 3.

Empirical Works: Purpose

- ▶ The purpose of this part is to provide a financial application to weigh the benefits and the costs of AMCSD vs. MCSD in financial data.
- ▶ Bali, Brown, and Caglayan (2013, MS) apply the *almost stochastic dominance* approach to test whether hedge funds outperform stocks and bonds. Their results from the realized and simulated return distributions indicate that the Long/Short Equity Hedge and Emerging Markets hedge fund strategies outperform the U.S. equity market, and the Long/Short Equity Hedge, Multi-strategy, Managed Futures, and Global Macro hedge fund strategies dominate the U.S. Treasury market.
- ▶ Thus, we would like to construct portfolios including hedge funds, stocks and bonds. We intend to show that using AMCSD can improve the efficiency of existing portfolios.

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- ▶ The hedge fund dataset is obtained from Hedge Fund Research (HFR) database.
- ▶ Between January 1994 and December 2011, out of the 18,720 hedge funds that reported monthly returns to HFR, we have 11,867 funds in the defunct/graveyard database and 6,853 funds in the live hedge fund database.
- ▶ The size of a fund is measured as the average monthly assets under management over the life of the fund.
- ▶ Based on our data, while the mean hedge fund size is \$149.5 million, the median hedge fund size is only \$28.2 million.

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Data: Screening

- ▶ We follow literature by including both live (6,853 funds) and dead funds (11,867) in our sample to eliminate survivorship bias.
- ▶ To avoid back-fill bias, we follow Fung and Hsieh (2000) and delete the first 12-month return histories of all individual hedge funds in our sample. Lastly, to address the multi-period sampling bias and to obtain sensible measures of risk for funds, we require that all hedge funds in our study have at least 24 months of return history (see Kosowski, Naik, and Teo (2007)) to mitigate the impact of multi-period sampling bias.
- ▶ After all these requirements we have 12,816 surviving and defunct funds in our sample (7,443 dead funds and 5,373 live funds).

- ▶ We compute the equal-weighted average returns of funds for each of the 7 investment styles reported in the HFR database to generate hedge fund indices; Emerging Markets, Equity Market Neutral, Event Driven, Fund of Funds, Macro, Relative Value and Equity Hedge.
- ▶ The performance of the U.S. equity market is measured by the S&P500 index returns and the performance of the short-term U.S. Treasury securities is presented by the 1-year Treasury returns.

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Table 1 Descriptive Statistics (in % except skewness and kurtosis)

| Asset | Mean | Median | Std Dev | Skewness | Kurtosis | Minimum | Maximum |
|------------------------------|--------|--------|---------|----------|----------|----------|---------|
| Macro | 0.8439 | 0.7433 | 2.0716 | 0.3655 | 0.0977 | -4.0632 | 7.2431 |
| Emerging Market | 1.0816 | 1.7414 | 4.8163 | -0.9066 | 4.6121 | -25.2822 | 17.7853 |
| Equity Hedge | 0.8872 | 1.1166 | 2.7629 | -0.4416 | 2.3616 | -10.8880 | 11.0608 |
| Event Driven | 0.8342 | 1.1006 | 1.8562 | -1.6426 | 5.9061 | -8.6978 | 4.6426 |
| Relative Value | 0.6880 | 0.8376 | 1.3100 | -2.9249 | 18.0144 | -9.0643 | 4.0106 |
| Fund of Fund | 0.4990 | 0.6229 | 1.6553 | -0.6570 | 3.2184 | -6.4261 | 5.9474 |
| Equity Market Neutral | 0.5883 | 0.5757 | 0.8654 | -0.1481 | 2.8234 | -3.4469 | 3.6825 |
| S&P500 | 0.5640 | 1.1194 | 4.5272 | -0.6416 | 0.9372 | -16.9425 | 10.7723 |
| 1-year T-bill | 0.3226 | 0.3158 | 0.2936 | 0.4620 | 0.3400 | -0.3310 | 1.3061 |

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Figure 2 Emerging Market Fund v.s. S&P500 index (10% in 1-year bond)

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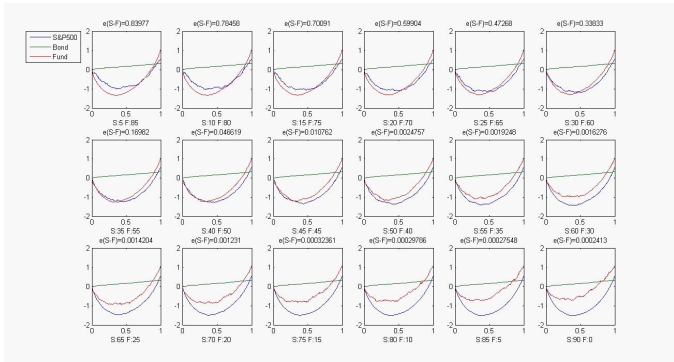


Figure 3 Macro Fund v.s. S&P500 index (10% in 1-year bond)

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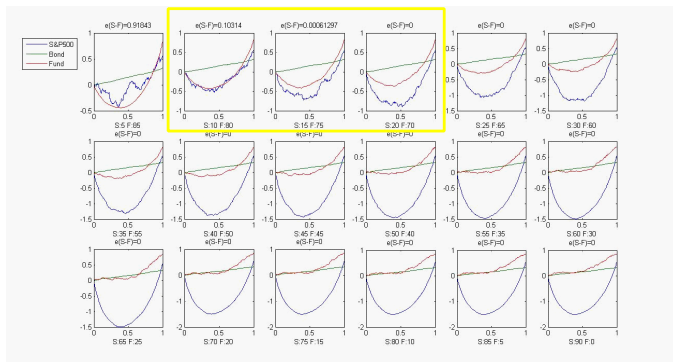


Figure 4 Macro Fund v.s. S&P500 index (10% in 1-year bond)

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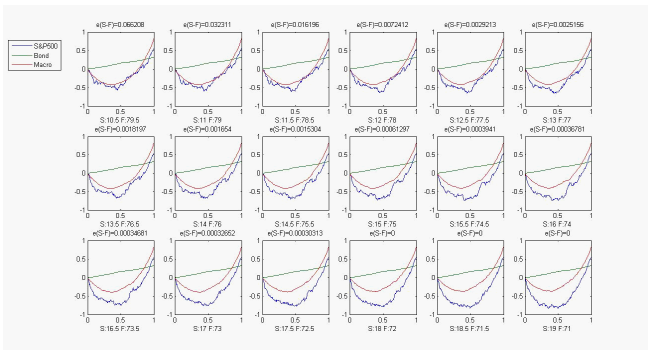
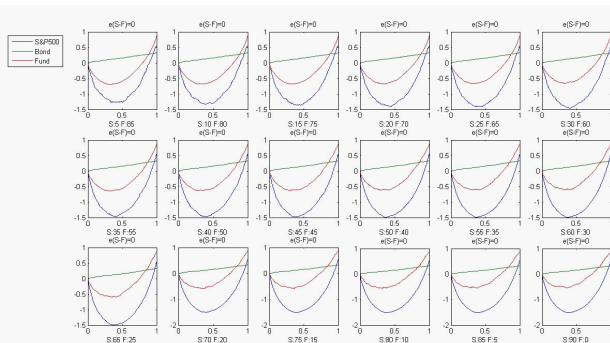


Figure 5 Equity Hedge Fund v.s. S&P500 index (10% in 1-year bond)



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Confined preferences

- ▶ The common preferences shared by all the decision-makers

$$U_N = \left\{ u \mid (-1)^{n+1} u^{(n)} \geq 0, n = 1, 2, \dots, N \right\}.$$

- ▶ Confined preferences:

$$U_N^*(\varepsilon_N) = \left\{ u \in U_N \mid \begin{array}{l} (-1)^{N+1} u^{(N)}(x) \\ \leq \inf \left\{ (-1)^{N+1} u^{(N)}(x) \right\} \left(\frac{1}{\varepsilon_N} - 1 \right) \quad \forall x \end{array} \right\}.$$

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Higher order AMCSD

- Now, starting from $B^{(1)}(t) = B(t)$, let us define iteratively for $n = 2, 3, \dots, N$

$$B^{(n)}(t) = \int_a^t B^{(n-1)}(s) ds,$$

$\Omega_n = \{t \in [a, b] : B^{(n)}(t) < 0\}$, and Ω_n^c as the complement of Ω_n in $[a, b]$.

Theorem

Given portfolio α , asset k dominates asset j for all individuals with preferences $u \in U_N^(\varepsilon_N)$, $N > 2$, if and only if*

$$\int_{\Omega_N} (-B^{(N)}(t)) dt \leq \varepsilon_N \int_a^b |B^{(N)}(t)| dt$$

and $B^{(n)}(b) \geq 0$, $n = 2, 3, \dots, N$.

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